Structural Diagrams

Mark Paral

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Contents

Variable Definitions

F	Force
V	Shear force
P	Axial force
R	Reaction force
M	Moment
T	Torque
p	Distributed load (per unit length)
b	Base dimension
h	Height dimension
r	Radius
L	Length
ΔL	Change in length
A	Area
ν	Poisson's ratio
E	Young's modulus
G	Shear modulus
Q	First moment of area / statical moment of area
I	Area moment of inertia / second moment of area
J	Second polar moment of area
k_s	Linear spring constant
k_t	Torsional spring constant
Δx	Linear displacement
θ	Angular displacement

1 Introduction

Real world aerospace problems can typically be simplified to basic structural concepts, such as an aircraft landing gear being represented as a simple beam and spring. This document lays out many of the basic structural concepts students might encounter on either homeworks or exams in an aerospace structures course. Here, we will cover topics from possible loading conditions to shear and moment diagrams, and hopefully everything in between.

2 Common Materials

There are a number of materials that are commonly used in aerospace engineering. In an effort to standardize the properties used, select common types will be listed here.

2.1 6061-T6 Aluminum

6061-T6 Aluminum is a commonly used aluminum alloy that also exhibits highly isotropic properties. The material properties are listed in Tab. 1

Material Property	**Value**
E	68.9 GPa
G	26 GPa
ν	0.33

Table 1: Material Properties for 6061-T6 Aluminum (via Aerospace Specification Metals Inc.)

3 Beams and Frames

There are two common setups for problems in structural mechanics: beams and frames.

3.1 Beams

A **Beam** (or bar) is a simple structural member that is used to resist moments and loads. It is subject to a multitude of loading and boundary conditions, but is only made up of a single structure typically depicted as a bar as in Fig. 1.

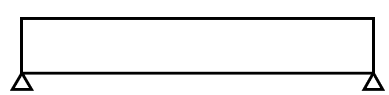

Figure 1: A Simple Beam Structure

3.2 Frames

A **Frame** is a structure made up of connected beams. An example of a frame is pictured in Fig. 2.

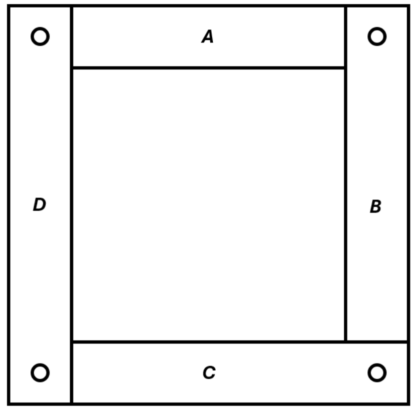

Figure 2: A Frame Structure

The connections between the bars are facilitated using joints, which allows for interactions between the beams to be considered (Newton's Third Law). For example, if we were to separately investigate the forces acting on beam **A**, we would note that equal and opposite forces must be acting on the rest of the truss structure. This is pictured in Fig. 3.

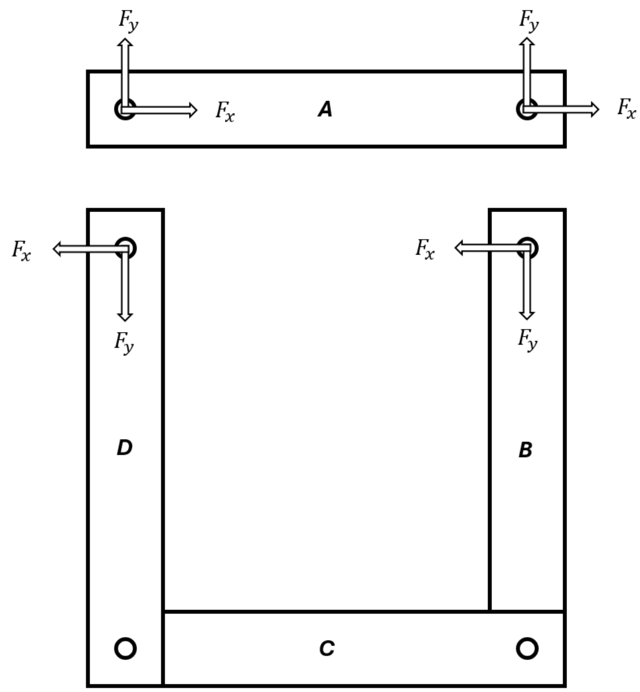

Figure 3: Internal Frame Forces

4 Loads

In many of the exercises you will run across in this class, an object (such as a beam) will be under loading from a multitude of forces or moments. This section will lay out the different loads you may encounter, and how one should handle them.

4.1 Point Force

A **Point Force** is a load applied at a specific point on an object. This is the simplest type of loading, as the force is absolute and the area of application is zero. An example of this is shown in Fig. 4.

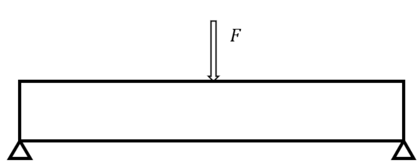

Figure 4: Point Force Example

4.2 Distributed Load

Distributed Loads are loads that are applied over an area instead of at a single point. These loads often have an associated formula describing the curvature of the loads. Additionally, there are simplifications that can be made when applying a load to a free body diagram (FBD).

Common distributed loads you might encounter are depicted in Fig. 5.

1. First, we have a **Constant Distributed Load**, represented as a rectangle in Fig. 5. The curvature of the constant distributed load is, as you might expect, constant. Therefore, the total force applied by the distributed load is given by Eq. 1, where L is the beam length and p is the force per unit length (constant).

$$F_p = pL \tag{1}$$

   This force is applied at the center of the constant distributed load as seen in Fig. 5

2. Second, we have a **Triangular Distributed Load**, represented by a triangle in Fig. 5. The curvature of the triangular distributed load is not constant, and varies in the $\hat{x}$ direction. In the case of Fig. 5, the maximum of the distributed load is on the left side with a value of p. The total force applied by the distributed load is given by Eq. 2, where L is the length of the beam.

$$F_p = \frac{1}{2}pL \tag{2}$$

   The force is applied at the centroid of the distributed load, or in the case of Fig. 5 at $\frac{L}{3}$.

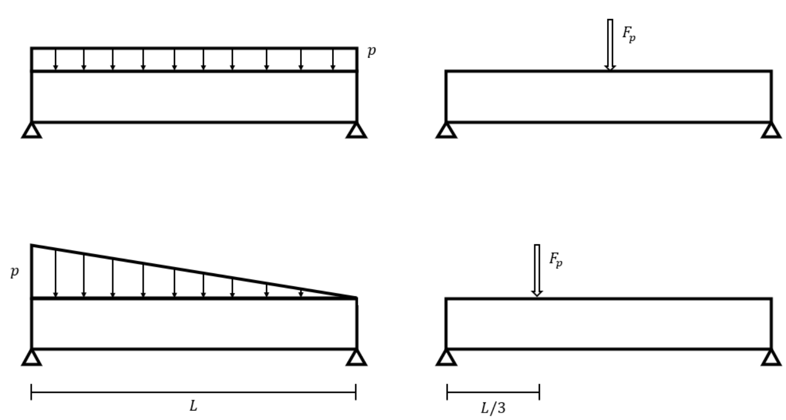

Figure 5: Distributed Loads and FBD Simplifications

4.3 Moment

Moments (M) are rotational loads caused by a force $\bar{F}$ acting at a distance $\bar{d}$ from an object's center of rotation. The formula for a moment is described by Eq. 3.

$$\bar{M} = \bar{d} \times \bar{F} \tag{3}$$

In this class, moments can result from forces or be applied directly themselves as seen in Fig. 6. Regardless, they will induce a rotation in the structure.

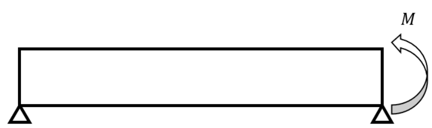

Figure 6: An Applied Moment

A **Torque** (T) is a specific type of moment which causes an object to rotate about one of its axes.

5 Joints

Structural objects do not typically float through empty space in practical application. Instead, they interact with the environment around them. Structures usually interact with the environment in a standardized way, via **Joints**. Joints represent the boundary conditions of the mechanical problem as they impose constraint on the allowable motion at the bounds of the problem we are considering.

There are a few common types of joints you should be aware of, as well as what they're respective boundary conditions look like.

1. First, there is the **Pin Joint** represented by the triangle in Fig. 7. The pin joint allows free rotation but restricts translation in the $\hat{x}$ and $\hat{y}$ directions. Because of this, reaction forces in the $\hat{x}$ and $\hat{y}$ directions must be considered.

2. Second, there is the **Roller** represented by the circle in Fig. 7. The roller allows free rotation and translational movement in one direction (in the example from Fig. 7 this is the $\hat{x}$ direction). However, the roller restricts translation in the direction perpendicular to the surface of the roller (in the example from Fig. 7 this is the $\hat{y}$ direction). In the case of this joint, reaction forces in the direction perpendicular to the surface must be considered.

3. Third, there is the **Fixed Support** or **Wall Attachment** represented by the horizontal line with diagonals in Fig. 7. The fixed support restricts both translational and rotational motion. Thus, reaction forces and moments must be considered.

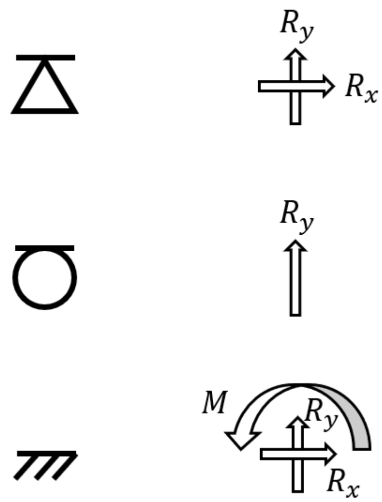

Figure 7: Joints and Corresponding Reactions

6 Springs

While often used as joints, **Springs** have unique properties that warrant special considerations. Springs exert a reaction force based off of the displacement they've undergone. This reaction force is in the opposite direction of the displacement, and is based off of a spring constant inherent to that spring. In essence, springs are boundary conditions dependent on displacement.

There are two primary types of springs you might encounter in this class.

1. First, there is the **Torsional Spring** represented as the spiral in Fig. 8. This spring exerts a reactionary moment based on the angular displacement (θ) it has undergone and its spring constant (k_t). This moment is given by Eq. 4.

$$M_s = -k_t\theta \tag{4}$$

2. Second, there is the standard **Linear Spring** represented by the zig-zag in Fig. 8. This spring exerts a reactionary force based on the displacement (Δx) it has undergone and its spring constant (k_s). This moment is given by Eq. 5.

$$F_s = -k_s \Delta x \tag{5}$$

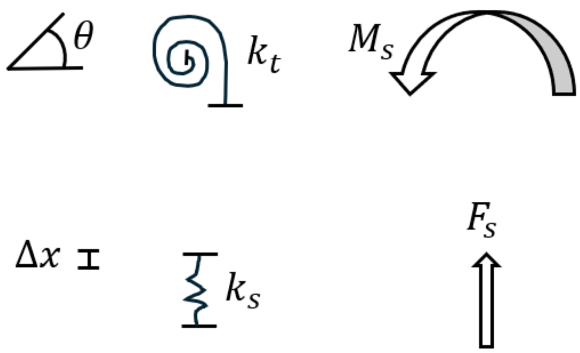

Figure 8: Spring Types

7 Free Body Diagrams

Free Body Diagrams (FBDs) are diagrams that simplify the model of a structural member into only the forces and moments acting on the object. You're likely familiar with these from introductory physics courses, and they remain a vital tool for understanding the performance and internal behaviors of structures.

The best way to demonstrate how to perform a FBD is with an example. Assume we are given a cantilever beam setup as seen in Fig. 9, where one end is fixed, the other is under an applied load F, and the entire beam is under a constant distributed load p. The corresponding FBD would simplify this diagram into Fig. 10, where all the external forces and moments are illustrated.

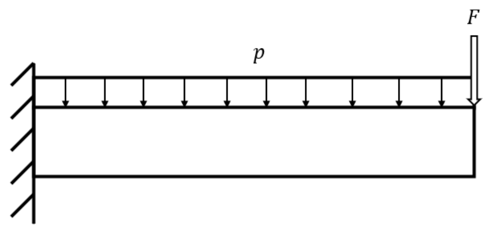

Figure 9: Cantilever Beam Problem

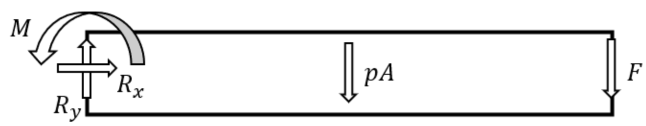

Figure 10: Cantilever Beam FBD

This may seem a bit rudimentary to some, but often the most important first step for any structural analysis problem is to sketch the FBD. This way you can be assured you are accounting for all the forces and moments during the remainder of the problem.

8 Shear and Moment Diagrams

A tool that will be important time and time again in this class are **Shear and Moment Diagrams**. Shear and moment diagrams allow you to calculate the internal shear and moments at any point along a structural element, such as a standard beam.

Let's say you want to determine when a beam might fail. In order to do that, you may need to know the maximum stress of the beam. The maximum stress of the beam, as noted in the previous sections, is often dependent on the maximum absolute moment in a beam. The shear and moment diagrams will illustrate that. This just one example of the usefulness of these diagrams.

Creating these diagrams can seem difficult, but so long as you follow a few rules they become very straight forward.

Applying Shear and Moment Cuts:

1. Start at one end of the object
2. Make a cut between the end and the next change in loading acting on the object
3. Draw the FBD including the internal shear force (V), internal axial force (P), and internal moment (M) using correct convention as shown in Fig. 11

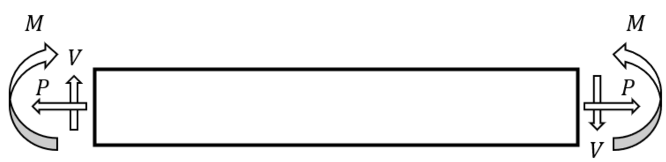

Figure 11: Internal Shear and Moment Convention

4. Calculate the value of the internal shear force, axial force, and moment dependent on x (the distance from the end of the object) using equilibrium equations
5. If there is another change in loading between the previous cut and the other end of the structure, return to step 2 and make a new cut. Otherwise, you're done cutting!
6. Sketch the curvature of the internal shear force and moment vs x

I understand that instructions can be convoluted and hard to understand, so let's walk through an example. Take a pinned beam under a point force F and constant distributed load per unit length p as depicted in Fig. 12. It's always important to sketch the FBD, so we do so in Fig. 13. Now let us start the process of sketching the shear moment diagrams.

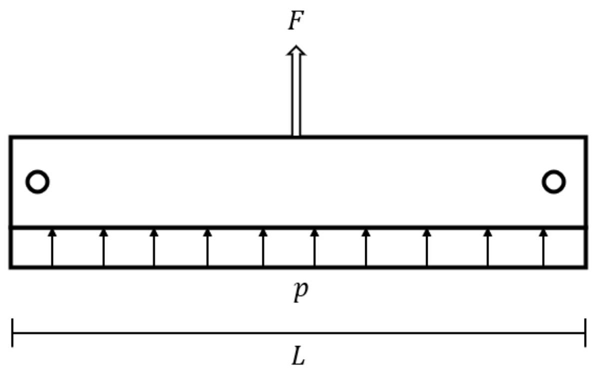

Figure 12: Shear and Moment Example Problem

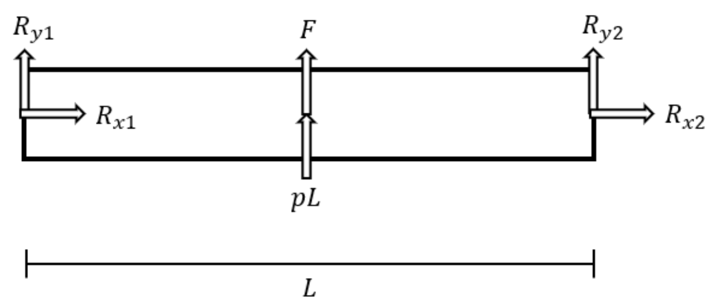

Figure 13: Shear and Moment Example FBD

I will choose the left end to start at. I want to make a cut before the applied force F (to adhere to the rules), so I will make my cut as seen in Fig. 14 (obeying the standard shear and moment convention). You will notice the distributed load is included in this diagram as it is dependent on x. I can derive the values for the internal shear (V), axial (P), and moment (M) using equilibrium equations as Eqs. 6, 7, and 8, respectively.

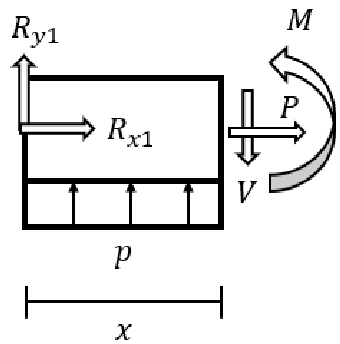

Figure 14: First Cut

$$V(x) = R_{y1} + px \tag{6}$$

$$P(x) = -R_{x1} \tag{7}$$

$$M(x) = R_{y1}x + p\frac{x^2}{2} \tag{8}$$

I'm not at the other end yet, so now I will make a cut after the applied force F as shown in Fig. 15. In this diagram, the internal shear, axial, and moment are defined as Eqs. 9, 10, and 11, respectively.

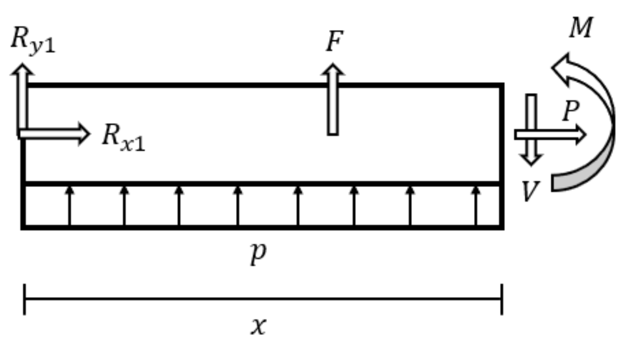

Figure 15: Second Cut

$$V(x) = R_{y1} + px + F \tag{9}$$

$$P(x) = -R_{x1} \tag{10}$$

$$M(x) = R_{y1}x + p\frac{x^2}{2} + F(x - \frac{L}{2}) \tag{11}$$

Finally, we are at the end of the beam. With the equations I've derived for the internal shear and moment, I can sketch plots given by Fig. 16 (assuming $F = 5$, $p = -5$, $L = 1$, and $R_{y1} = 0$).

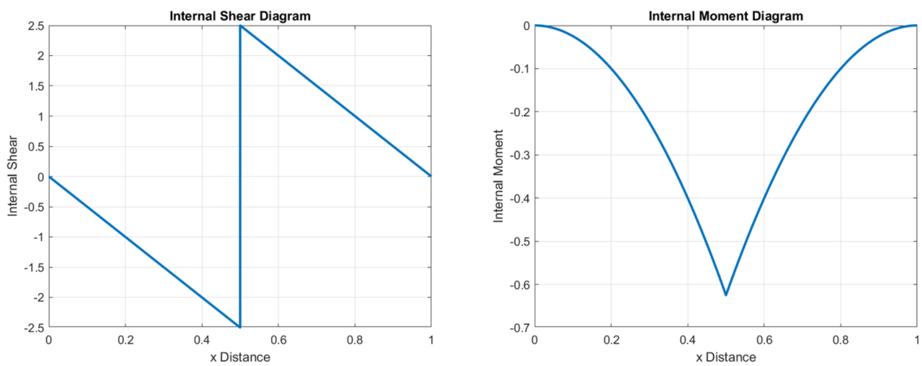

Figure 16: Shear and Moment Diagrams

It's important to take a moment to interpret the meaning of these plots. For example, we will note that the maximum absolute internal moment appears at $L/2$. As stated before, this kind of information will become particularly valuable for failure analysis in the future.

A MATLAB Codes

```
% AA240
% Structural Diagrams Code - Shear and Moment Diagrams
% Author(s): Mark Paral

% Clear workspace
clc
clear all
close all

% Resolution variable
count = 100;

% Define problem parameters
L=1;
F=5;
P=-5;
w=1;
Ry1=0;
x1 = linspace(0,L/2,count);
x2 = linspace(L/2,L,count);
x = cat(2,x1,x2);

% Shear and moment at first cut
V1 = Ry1+P.*w.*x1;
M1 = Ry1.*x1+P.*w.*x1.^2./2;

% Shear and moment at second cut
V2 = Ry1+P.*w.*x2+F;
M2 = Ry1.*x2+P.*w.*x2.^2./2+F.*(x2-L/2);

% Combine shear and moment
V = cat(2,V1,V2);
M = cat(2,M1,M2);

% Plot shear
figure;
plot(x,V,'LineWidth',2);
title("Internal Shear Diagram")
ylabel("Internal Shear");
xlabel("x Distance");
grid on;

% Plot moment
```

```
figure;
plot(x,M,'LineWidth',2);
title("Internal Moment Diagram")
ylabel("Internal Moment");
xlabel("x Distance");
grid on;
```